Axioms of Arithmetic

Given: $a \in \mathbb{R}$, $b \in \mathbb{R}$, $c \in \mathbb{R}$, where $\mathbb{R}$ = {Real Numbers}, then:

	Addition	
1A	$a+b=b+a$	Commutativity
2A	$(a+b)+c=a+(b+c)$	Associativity
3A	$a+(-a)=(-a)+a=0$	Inverse Property
4A	$a+0=0+a=a$	Identity Element
5A	$a+b=c$	Closure
6A	$a-b=a+(-b)$	Subtraction (*def.*)

	Multiplication	
1M	$a\times b=b\times a$	Commutativity
2M	$(a\times b)\times c=a\times(b\times c)$	Associativity
3M	$a\times\frac{1}{a}=\frac{1}{a}\times a=1,\ a\neq 0$	Inverse Property
4M	$a\times 1=1\times a=a$	Identity Element
5M	$a\times b=c$	Closure
6M	$a/b=a\times(1/b),\ b\neq 0$	Division (*def.*)

7	$a\times 0=0\times a=0$	Multiplication Property of 0
8	$a\times -1=-1\times a=-a$	Multiplication Property of −1
9	$-(-a)=a$	Axiom of Opposites
10	$a(b+c)=ab+ac$	Distributive Property
11	$-(a+b)=(-a)+(-b)$	Property of Opposite of a Sum
12	$(-a)b=a(-b)=-ab$ and $(-a)(-b)=ab$	Property of Opposites in Products
13	$\frac{1}{\left(\frac{1}{a}\right)}=a$ and $\frac{1}{-a}=-\frac{1}{a},\ a\neq 0$	Axiom of Reciprocals
14	$\frac{1}{ab}=\frac{1}{a}\times\frac{1}{b}$	Property of the Reciprocal of a Product

15	$a=a$	Reflexive Property of Equality
16	If $a=b$, then $b=a$	Symmetrical Property of Equality
17	If $a=b$ and $b=c$, then $a=c$	Transitive Property of Equality

18	If $a=b$, then $a+c=b+c$	Addition Property of Equality
19	If $a=b$, then $a-c=b-c$	Subtraction Property of Equality
20	If $a=b$, then $ac=bc$	Multiplication Property of Equality
21	If $a=b$ and $c\neq 0$, then $\frac{a}{c}=\frac{b}{c}$	Division Property of Equality

22	For all real numbers *a* and *b*, one and only one of the following statements is true: $a < b$, $a = b$, $a > b$.	Axiom of Comparison
23	Every decimal represents a real number and every real number has a decimal representation.	Axiom of Completeness
24	Between any two real numbers there is another real number.	Property of Density

Properties A

Given: $a \in \mathbb{R}$, $b \in \mathbb{R}$, c $\in \mathbb{R}$, where $\mathbb{R}$ = {Real Numbers}, then:

I.A.1	$a+b=b+a$	Commutative Property of Addition
I.A.2	$a+(b+c)=(a+b)+c$	Associative Property of Addition
I.A.3	$a+(-a)=(-a)+a=0$	Additive Inverse Property
I.A.4	$a+0=0+a=a$	Additive Identity Element
I.A.5	$a+b=c$	Closure for Addition
I.M.1	$a\times b=b\times a$	Commutative Property of Multiplication
1.M.2	$(a\times b)\times c=a\times(b\times c)$	Associative Property of Multiplication
1.M.3	$a\times\frac{1}{a}=\frac{1}{a}\times a=1,\ a\neq 0$	Multiplicative Inverse Property
1.M.4	$a\times 1=1\times a=a$	Multiplicative Identity Element
1.M.5	$a\times b=c$	Closure for Multiplication
1.M.6	$a\times 0=0\times a=0$	Multiplication Property of Zero
II.1	$a=a$	Reflexive Property of Equality
II.2	If $a=b$, then $b=a$	Symmetrical Property of Equality
II.3	If $a=b,\ b=c$, then $a=c$	Transitive Property of Equality
III	$a(b+c)=ab+ac$	Distributive Property
IV	$-(-a)=a$	Axiom of Opposites
V	$-(a+b)=(-a)+(-b)$	Property of Opposite of a Sum
VI	$a\times -1=-1\times a=-a$	Multiplication Property of −1
VII	$(-a)(b)=a(-b)=-ab$ and $(-a)(-b)=ab$	Property of Opposites in Products
VIII	$\frac{1}{\left(\frac{1}{a}\right)}=a$ and $\frac{1}{-a}=-\frac{1}{a},\ a\neq 0$	Axiom of Reciprocals
IX	$\frac{1}{ab}=\frac{1}{a}\times\frac{1}{b}$	Property of the Reciprocal of a Product
X	$a-b=a+(-b)$	Definition of Subtraction
XI	$\frac{a}{b}=\frac{1}{b}\times a=a\times\frac{1}{b},\ b\neq 0$	Definition of Division
XII	If $a=b$, then $a+c=b+c$	Addition Property of Equality
XIII	If $a=b$, then $a-c=b-c$	Subtraction Property of Equality
XIV	If $a=b$, then $ac=bc$	Multiplication Property of Equality
XV	If $a=b,\ c\neq 0$, then $\frac{\mathrm{a}}{\mathrm{c}}=\frac{b}{c}$	Division Property of Equality
XVI	For all real numbers a and b, one and only one of the following statements is true: $a<b$, $a=b$, $a>b$.	Axiom of Comparison
XVII	Every decimal represents a real number and every real number has a decimal representation.	Axiom of Completeness
XVIII	Between any two real numbers there is another real number	Property of Density

Adapted by Cary Millsap from James R. Harkey “Properties A” (1976). Visit *http://carymillsap.blogspot.com*.

CVMillsap PROPERTIES A

Given: $a \in R$, $b \in R$, $c \in R$ where R = {Real Numbers}, then:

I Addition

1. $a+b=b+a$ Commutative Property of Addition
2. $a+(b+c)=(a+b)+c$ Associative Property of Addition
3. $a+(-a)=(-a)+a=0$ Additive Inverse Property
4. $a+0=0+a=a$ Additive Identity Element
5. $a+b=c$ Closure for Addition

Multiplication

1. $a \cdot b = b \cdot a$ Commutative Property of Multiplication
2. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ Associative Property of Multiplication
3. $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$, $a \neq 0$ Multiplicative Inverse Property
4. $a \cdot 1 = 1 \cdot a = a$ Multiplicative Identity Element.
5. $a \cdot b = c$ Closure for Multiplication
6. $a \cdot 0 = 0 \cdot a = 0$ Multiplication Property of Zero

II Axioms of Equality:
1. $a=a$ Reflexive Property of Equality.
2. If $a=b$, then $b=a$. Symmetrical Property of Equality
3. If $a=b$, $b=c$, then $a=c$. Transitive Property of Equality.

III Distributive Property: $a[b+c]=ab+ac$

IV Axiom of Opposites: $-(-a)=a$

V $-(a+b)=(-a)+(-b)$ Property of Opposite of a Sum

VI $a \cdot (-1) = (-1) \cdot a = -a$ Multiplication Property of -1

VII $(-a)(b) = a(-b) = -ab$ and $(-a)(-b) = ab$ Property of Opposites in Products.

VIII $\frac{1}{\frac{1}{a}} = a$ and $\frac{1}{-a} = -\frac{1}{a}$, $a \neq 0$. Axiom of Reciprocals

IX $\frac{1}{ab} = \frac{1}{a} \cdot \frac{1}{b}$ Property of the Reciprocal of a Product.

X $a-b=a+(-b)$ Definition of Subtraction.

XI $\frac{a}{b} = \frac{1}{b} \cdot a = a \cdot \frac{1}{b}$, $b \neq 0$ Definition of Division

XII If $a=b$, then $a+c=b+c$ Addition Property of Equality

XIII If $a=b$, then $a-c=b-c$ Subtraction Property of Equality

XIV If $a=b$, then $ac=bc$ Multiplication Property of Equality

XV If $a=b$, $c \neq 0$, then $\frac{a}{c}=\frac{b}{c}$ Division Property of Equality.

XVI For all Real Numbers a and b, one and only one of the following statements is true: $a<b$, $a=b$, $a>b$ Axiom of Comparison.

XVII Every decimal represents a real Number and every real Number has a decimal representation. Axiom of Completeness.

XVIII Between any two real Numbers there is another real Number. Property of Density.